Turán number for disjoints copies of paths

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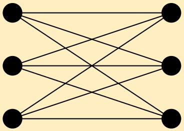

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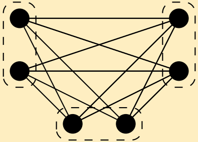

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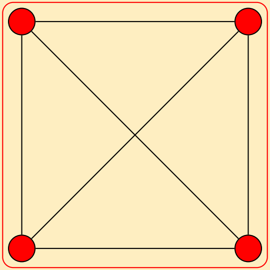
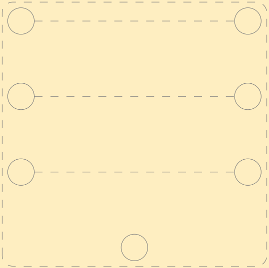
- Goal is to beat this result for certain bipartite graphs.

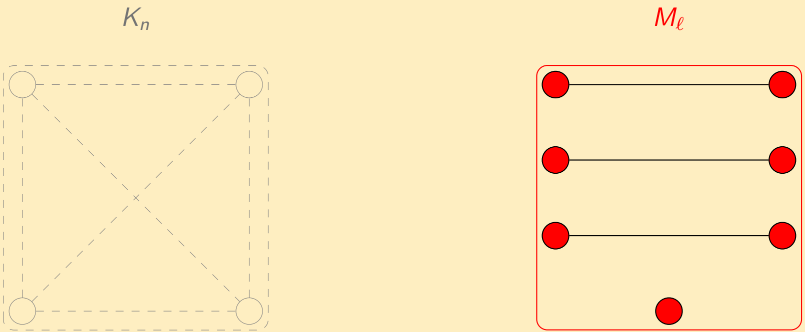
Complete graph

K_n



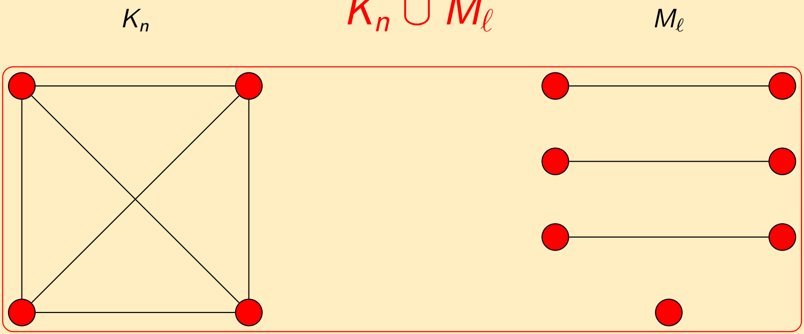


Matching graph



Union

$K_n \cup M_\ell$

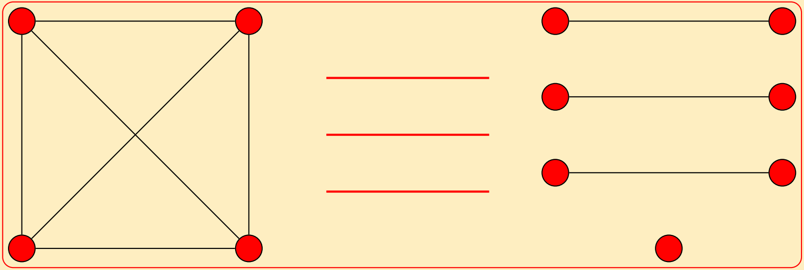


kG is the graph formed by the union of k disjoint copies of G.

Join

$$K_n \vee M_\ell$$

K_n M_ℓ



Easy for some small graphs

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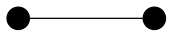

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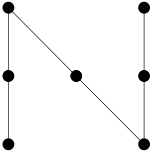

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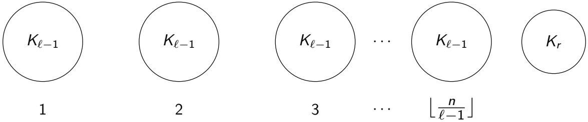

R. Faudree and R. Schelp.
Path ramsey numbers in multicolorings.
Journal of combinatorial theory series B, 1975

Turán Number for disjoint copies of paths

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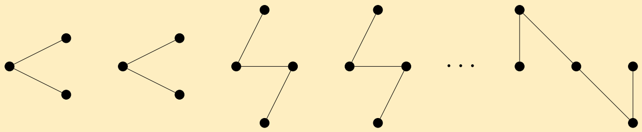

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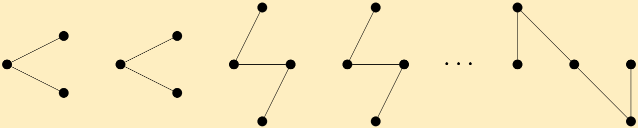

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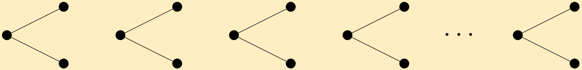

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Combinatorics, Probability and Computing, 2011

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B. Lidický, H. Liu, C. Palmer.
On the Turán number of forests.
arXiv:1204.3102, 2012

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P. Erdős and T. Gallai
On maximal paths and circuits of graphs
Acta Mathematica Academiae Scientiarum Hungaricae, 1959

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- $\mathcal{P} = kP_3$ (our result).

V. Campos and R. Lopes
A proof for a conjecture of Gorgol.
Electronic Notes in Discrete Mathematics, 2015

Disjoint copies of P_3

Gorgol (2011)

$\mathrm{ex}(n, kP_3) \geq \mathrm{Gorgol}(n, k)$

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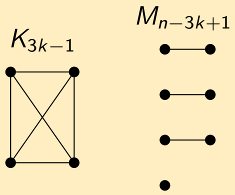

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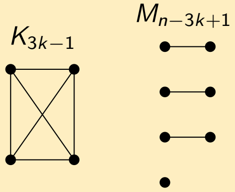

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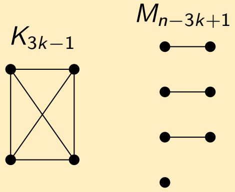

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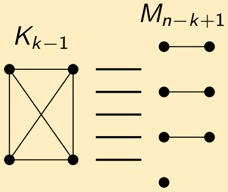

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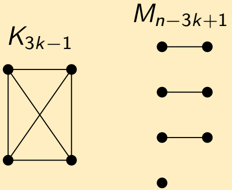

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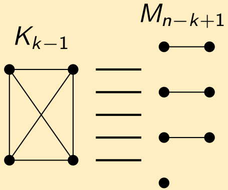

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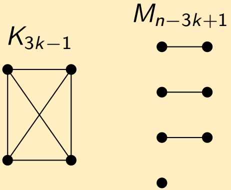

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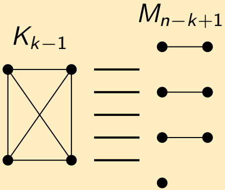

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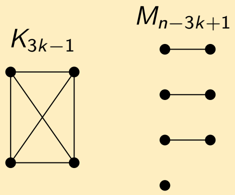

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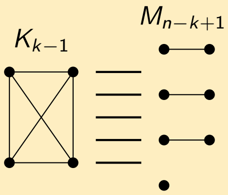

Related results

Sharp for:

- $k = 2, 3$

I. Gorgol
Turán Numbers of disjoints copies of graphs.
Graphs and Combinatories (2011).

Related results

Sharp for:

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Our result: Proof of Gorgol's Conjecture

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Builds a collection $\mathcal{Q} = \{Q_1 \cdots Q_k\}$ where:

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- $G[Q_i]$ contains P_3.

Algorithm

Input:

Overview:

Improvement condition:

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Algorithm

$\longrightarrow$: improvement not found.
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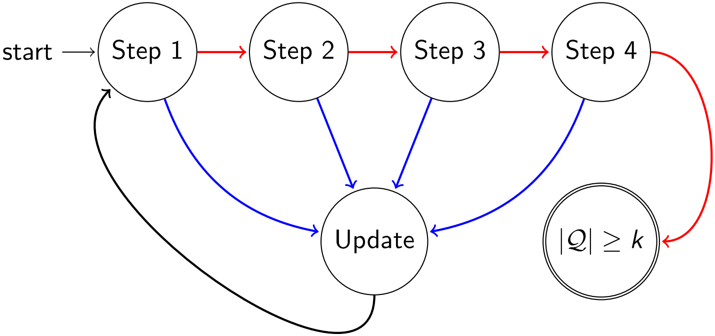
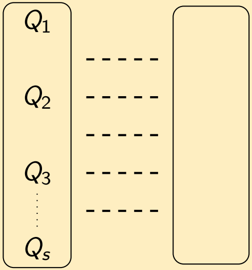


Iteration

Given $\mathcal{Q} = \{Q_1, \cdots, Q_s\}$, $s < k$

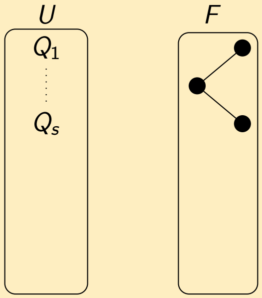
Dividing the graph

$$U = \bigcup_i V(Q_i) \qquad F = V(G) - U$$



Iteration: Step 1

Find P_3 in F



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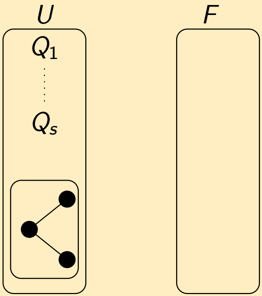
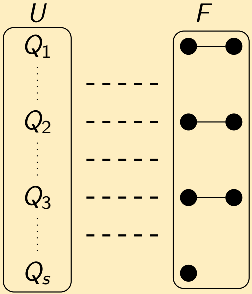


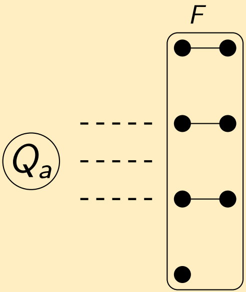
Iteration: Step 1

After Step 1



Overview of steps 2-4

Step 2

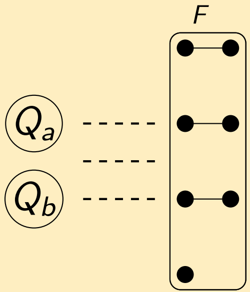


Local improvements

- $Q_a \cup F$.

Overview of steps 2-4

Step 3

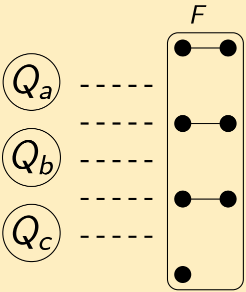


Local improvements

- $Q_a \cup F$.
- $Q_a \cup Q_b \cup F$.

Overview of steps 2-4

Step 4

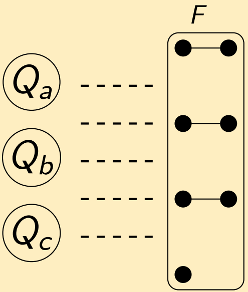


Local improvements

- $Q_a \cup F$.
- $Q_a \cup Q_b \cup F$.
- $Q_a \cup Q_b \cup Q_c \cup F$.

Overview of steps 2-4

Step 4



Local improvements

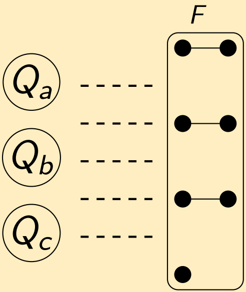
- $Q_a \cup F$.
- $Q_a \cup Q_b \cup F$.
- $Q_a \cup Q_b \cup Q_c \cup F$.

Naive complexity

- $O(n^{12}k^3)$.

Overview of steps 2-4

Step 4



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- $O(n^{12}k^3)$.

Theorem

If no local improvements are found, then $|\mathcal{Q}| \geq k$.

Time complexity

- General improvements might be slow to compute.

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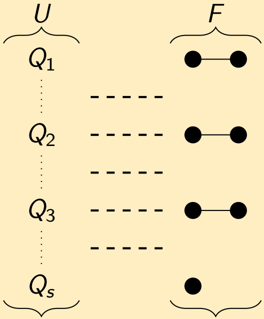
Time complexity

- General improvements might be slow to compute.
- Can prove the theorem with a small set of local improvements.
- Algorithm complexity: $O(k|E|)$.
- Amortized $O(|E|)$ time to find each copy of P_3.

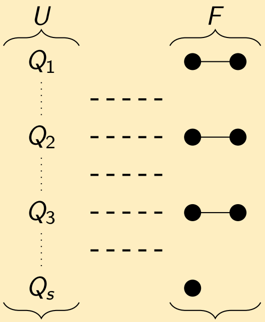
Proof overview

G after Step 1

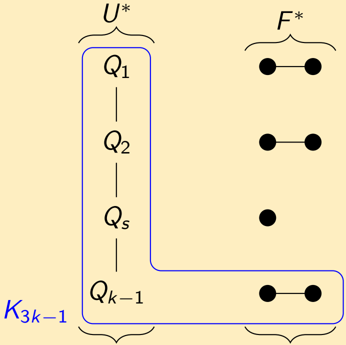


Proof overview

G after Step 1



$G^* = K_{3k-1} \cup M_{n-3k+1}$



Proof overview

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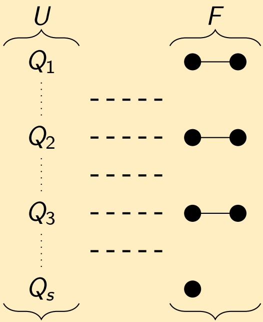

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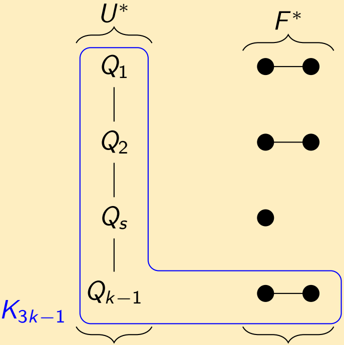
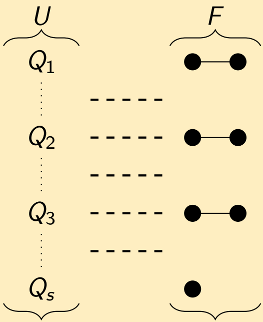


After Step 1 no longer applies

Proof overview

G after Step 1



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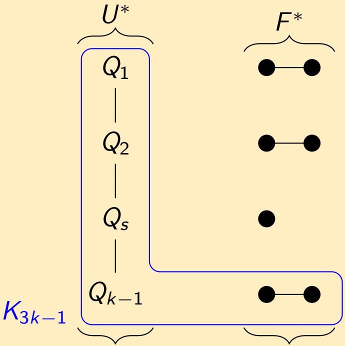
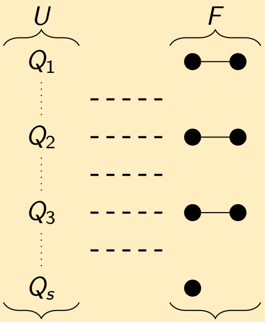


After Step 1 no longer applies

- We compare G and G^*.

Proof overview

G after Step 1



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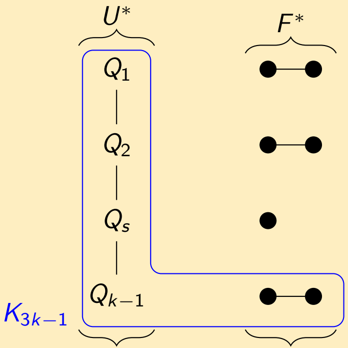


After Step 1 no longer applies

- We compare G and G^*.
- As $\mathrm{e}(G) > \mathrm{e}(G^*)$, we show that $\mathrm{e}(Q_i) + \mathrm{e}(Q_i, F) - 9 \geq 1$ for some $Q_i \in \mathcal{Q}$.

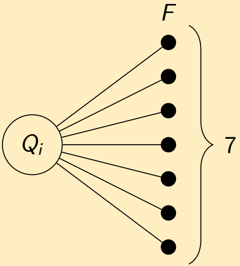
Proof Overview

$$\mathrm{e}(Q_i) + \mathrm{e}(Q_i, F) - 9 \geq 1 \implies \left\{ \right.$$

Proof Overview

$$\mathrm{e}(Q_i) + \mathrm{e}(Q_i, F) - 9 \geq 1 \implies \begin{cases} \mathrm{e}(Q_i, F) \geq 7 \\ \end{cases}$$

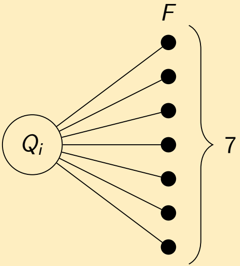
$\mathrm{e}(Q_i, F) \geq 7$



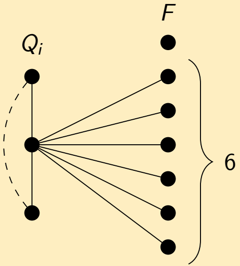
Proof Overview

$$\mathrm{e}(Q_i) + \mathrm{e}(Q_i, F) - 9 \geq 1 \implies \begin{cases} \mathrm{e}(Q_i, F) \geq 7 \\ \exists x_i \in Q_i : d(x_i, F) \geq 6 \end{cases}$$

$\mathrm{e}(Q_i, F) \geq 7$



$d(x_i, F) \geq 6$



Proof Overview: Step 3

$e(Q_i, Q_j) \geq 6$

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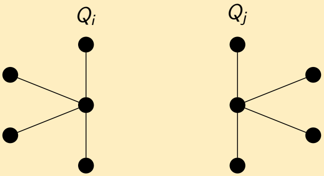

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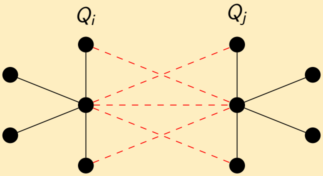

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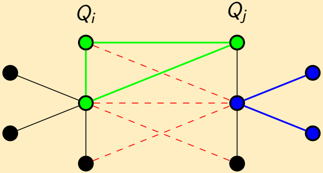

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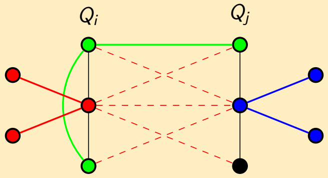

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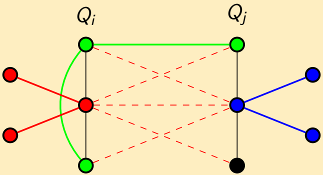
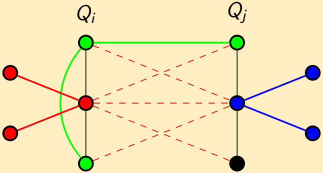


- If Step 3 no longer applies, then $e(Q_i, Q_j) \leq 5$ for all pairs Q_i, Q_j with excess edges.

Proof Overview: Step 3

$e(Q_i, Q_j) \geq 6$



- If Step 3 no longer applies, then $ne(Q_i, Q_j) \geq 4$ for all pairs Q_i, Q_j with excess edges.

Properties from the Steps

After Step 1

- Some sets Q_i have many edges to vertices in F.

Properties from the Steps

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$$\sum_{i \leq s} (\mathrm{e}(Q_i) + \mathrm{e}(Q_i, F) - 9) > \sum_{1 \leq i < j \leq s} \mathrm{ne}(Q_i, Q_j).$$

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After Steps 2, 3 and 4

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After Steps 2, 3 and 4

- $\sum_{1 \leq i < j \leq s} \mathrm{ne}(Q_i, Q_j) \geq g(k)$

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- Some sets Q_i have many edges to vertices in F.

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After Steps 2, 3 and 4

- $\sum_{1 \leq i < j \leq s} \mathrm{ne}(Q_i, Q_j) \geq g(k)$
- $\sum_{i \leq s} (\mathrm{e}(Q_i) + \mathrm{e}(Q_i, F) - 9) \leq f(k)$

Properties from the Steps

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- Some sets Q_i have many edges to vertices in F.

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- $\sum_{1 \leq i < j \leq s} \mathrm{ne}(Q_i, Q_j) \geq g(k)$
- $\sum_{i \leq s} (\mathrm{e}(Q_i) + \mathrm{e}(Q_i, F) - 9) \leq f(k)$

We prove the theorem by showing that if the algorithm stops before k copies of P_3 are found, then $g(k) \geq f(k)$ and a contradiction is met.

Future works

- k bigger stars.
- Count the number of graphs on n vertices that are free of kP_3.
- Stability.

Thank you.

Questions?